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13. ABSTRACT (Maximum 200 words)

This report results from a contract tasking Moscow State Institute of Radio Engineering as follows: develop new concepts of gamma-ray lasing alternative to the traditional approaches based on the Moessbauer technique.

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FINAL REPORT

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European Office of Aerospace Research and Development
on the Project SPC-96-4032 (Contract F61708-96-W0199)

Study of Doppler-free two-quantum induced gamma emission

May 15, 1997

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Objectives and scope of the project

In this project we planned to develop some new concepts of gamma-ray lasing alternative to the traditional approaches based on the Moessbauer technique. Two separate branches we intended to discuss: (I) two-quantum induced radiative transitions in an ensemble of free excited nuclei and (II) two-quantum induced annihilation of electrons and positrons. Technical objective of the project was to work out a concrete scheme for the quantum mechanical calculations of two-quantum Doppler-free induced gamma emission (IGE) of free excited nuclei or antiparticles.

Achievements

The main result of the theoretical research of the project is the development of the method of external ignition of two-quantum IGE of (I) free excited nuclei or (II) free positronium atoms by counterpropagating intense photon beams. The performed analysis reveals the following advantages of this method:

- In contrast to one-quantum emission with Doppler-broadened line, all nuclei regardless of their random individual velocities turn out to be involved in the process of two-quantum IGE of photons with the energies near to a half of the nuclear transition energy (only for (I) free excited nuclei).

- A specific dynamic distributed feedback inherent to two-quantum emission in conterpropagating photon beams only is settled without any reflecting structures (for both (I) free excited nuclei and (II) free positronium atoms).
- Nonlinearity of the feedback leads to release of nuclear storage energy or positronium annihilation energy in avalanche-like manner accompanied by emission of a giant pulse of coherent gamma quanta (for both (I) free excited nuclei and (II) free positronium atoms).
- Relativistic motion of positronium atoms (particles with relatively small mass) permits us to lower the requirements to frequency, angular divergence and flux density of one of the counterpropagating igniting photon beams. The other beam could appears due to spontaneous-stimulated annihilation transitions caused by the first igniting beam since the spontaneous photons in this case proves to be perfectly matched both in frequency and direction.(for (II) free positronium atoms).

Technical Background

As is well known, the cross section for induced emission is rapidly decreased in high frequency range because of both the decrease of the emission wave length and the increase of the Doppler line width which is proportional to transition frequency. Therefore, observing of IGE seems to be unlikely without radical suppression the Doppler broadening of emission line. In the most proposals for gamma-ray lasing the Moessbauer recoilless transitions of nuclei in solids are suggested to use in order to avoid thermal broadening of emission line and, thus, to increase the IGE cross section. However, the lack of success of long standing efforts to design a self-consistent scheme for obtaining gamma-ray lasing in solids [1-4] impels us to search new approaches alternative to the traditional schemes based on the Moessbauer technique.

Two separate non-Moessbauer approaches to the problem of gamma-ray lasing have been previously discussed in literature: induced gamma emission of monokinetized nuclear beams [5, 6] and coherent gamma emission by induced annihilation of electron-positron pairs or positronium atoms [7-14].

Reported here is success in:

(I) theoretical study of an alternative way to remove the pernicious influence of chaotic motion of free nuclei by means of external ignition of two-quantum IGE process in

counterpropagating intense photon beams. This approach, first proposed in [15], uses all the experience gained in two-quantum sub-Doppler optical absorption spectroscopy [16].

(II) theoretical study of the process of external ignition of two-quantum induced annihilation of free positronium atoms by counterpropagating intense photon beams.

Report

(I) *External ignition of two-quantum IGE of free nuclei.*

According to the lows of energy and momentum conservation, any nucleus regardless of its velocity is capable to absorb (or emit) simultaneously two quanta with opposite directions of wave vectors and with the same energies equal to a half of the nuclear transition energy $E_0 = \hbar\omega_0$. The Doppler shifts to the frequencies of such quanta are equal in value but opposite in sign. Therefore, a motion of nuclei can not rule out the sum of the quanta energies beyond the resonance with the nuclear transition energy. Thus, the spectral distribution of two-quantum IGE under condition of external ignition by counterpropagating beams of photons of equal energies will features a narrow peak near to a half of the transition energy. This peak is associated with contribution to IGE of all nuclei regardless of their random individual velocities and it is in contrast with the background formed by the emission of particular groups of nuclei belonging to different parts of their velocity distribution.

The steady-state amplification of the counterpropagating beams with photon flux densities I and I^* [$cm^{-2}s^{-1}$] within the spectral line width of the nuclear transition $\Delta\omega_0$ around the frequency $\omega_0/2$ is governed above the ignition threshold by the equations [17]

$$\frac{dI}{dz} = -\frac{dI^*}{dz} = \beta(n_2 - n_1)II^* \tag{1}$$

where the rate constant for induced gamma emission β is estimated by the expression

$$\beta = \left(\frac{8\pi}{3}\right)^2 \frac{\alpha^2 a^4/\Delta\omega_0}{\left(1 - 2E_s/E_0\right)^2} \tag{2}$$

and E_s is the energy of the neighboring intermediate nuclear level situated between the levels involved in the considered nuclear transition, a - is the radius of a nucleus, α - is the fine

structure constant. The inverted population difference $(n_2 - n_1)$ is expressed through the initial population difference n_0 by the relation

$$n_2 - n_1 = \frac{n_0}{1 + II^*/I_s^2}, \tag{3}$$

where I_s is the saturation photon flux density.

A positive gain with $dI/dz > 0$ and $-dI^*/dz > 0$ is achieved under the following condition

$$I_{ign} > \frac{\sigma n}{\beta n_0} \tag{4}$$

which can be regarded as a threshold condition for ignition and σ - is the cross section for photon scattering from igniting beams, n - is the total concentration of nuclei.

Integration of equations (1) with allowance for the saturation of excessive population (3) yields the transcendent equation for the net output photon flux density I_n on the length L :

$$2\left(\frac{I_n}{I_s} + \frac{2I_{ign}}{I_s}\right)^{-1} \ln\left(1 + \frac{I_n}{I_{ign}}\right) + \frac{I_n}{I_s} = \beta n_0 I_s L\ , \tag{5}$$

where $I_{ign} = I^*_{ign}$ is the igniting photon flux densities at the input of the gain region.

Figure 1 displayed the dependence of normalized net output photon flux density I_n/I_s versus the medium activity $A = \beta n_0 I_s L$, found from the equation (5), with the squared ratio $\left(I_{ign}/I_s\right)^2$ taken as a parameter of a family of curves. These curves reveal an ambiguous S-like behavior at a high enough medium activity . As the parameter A reaches its critical value $A_{cr} = (\beta n_0 I_s L)_{cr} \sim 1$, the output photon flux density is switched to the upper branch of the S-like curve. This process is accompanied by abrupt avalanche-like devastation of the population of upper level, which give rise to a burst generation of a giant pulse of coherent gamma quanta.

Discontinuities in dependencies shown in figure 1 are due to the dynamic distributed feedback, which arises in induced two-quantum emission in counterpropagating photon beams

without any reflecting structures, creating of which in gamma frequency range is a very complicated task. Indeed, due to intrinsic nonlinearity of two-quantum emission the counterpropagating waves of IGE proves to be perfectly matched in phase and coupled to each other. This coherent coupling arising in each event of two-quantum IGE can be regarded as a some kind of positive distributed feedback, which leads to forming a standing wave in the amplification region (the main attribute of a feedback) in the absence of any mirrors or periodic scattering structures.

Conventional distributed feedback based on a stationary periodic scattering structure is characterized by a coefficient of coupling ρ of counterpropagating waves. This coupling coefficient is defined as a variation in the photon flux density dI^* in the backward wave within length element dz divided by the flux density I in the forward wave,

$$\rho = \frac{1}{I}\frac{dI^*}{dz}. \tag{6}$$

In an induced two-quantum process in the field of counterpropagating waves, the photon flux density in the backward wave changes in each event of stimulated emission. Because of intrinsic features of induced emission, newly created photons are perfectly phase-matched and are emitted into an appropriate mode. Therefore, according to (1), the nonlinear coefficient of dynamic distributed feedback in induced two-quantum emission is given by

$$\rho = \beta(n_2 - n_1)I^* \tag{7}$$

and increases proportionally to I^*. Because of the nonlinearity of the feedback excitation of nuclei is released in an avalanche-like manner, which is accompanied by emission of a giant pulse of gamma quanta.

Numerical estimates are given for a hypothetical "fortunate" nucleus with A=150, a transition energy $E_0 = 10^5$eV and an intermediate energy level with a detuning $|1-2E_s/E_0|$=2.10^{-6}. For such a detuning, expression (2) give the following estimate for the constant of stimulated emission β = $(2\cdot10^{-40}$ cm$^4)/\Delta\omega_0$. In accordance with (4), the threshold spectral flux density of igniting photons can be estimated as $I_{ign}/\Delta\omega_0 = 3\cdot10^{18}$ cm^{-2} , where we assumed that $n_0 = 0.6n$ and took into account that the scattering of gamma

quanta with an energy of about 50keV mainly occurs through photoionization of atoms with the scattering cross section $\sigma = 6 \cdot 10^{-22}$ cm^2.

The igniting photon flux density at a critical point is estimated as $I_{ign}/I_s \sim 1/(\beta n_0 I_s L)_{cr} \sim 1$, which can be satisfied at the threshold spectral photon flux density $I_{ign}/\Delta\omega_0 = 3 \cdot 10^{18}$ cm^{-2} with, for example, $n_0 = 10^{17}$ cm^{-3} and $L = 10^4$ cm .

Finally, let us compare the estimated threshold spectral photon flux density in the igniting beams with the capabilities of the available sources of gamma radiation. The spectral density of the photon flux in synchrotron radiation (within a solid angle of 10^{-5}steradian) is estimated as approximately 10cm^{-2} [18], which is many orders of magnitude lower than the required value of $I_{ign}/\Delta\omega_0$. Although x-ray lasers ensure a higher spectral density of the photon flux, about 10^{15}cm^{-2} [18], such a spectral density of photon flux is lower than the required one by three orders of magnitude. In addition, the pulse duration of radiation produced by x-ray lasers is not sufficient to ensure ignition.

The performed analysis reveals the main advantages and drawbacks of the method of external ignition of induced two-quantum emission from free excited nuclei by counterpropagating photon beams, which can be summarized as follows:

- In contrast to single-quantum emission in an ensemble of nuclei with Doppler-broadened gain line, induced two-quantum emission of gamma photons caused by counterpropagating igniting beams involves all nuclei regardless of their individual velocities.
- A specific dynamic distributed feedback, which is characteristic of induced two-quantum emission in counterpropagating photon beams only, is established in the absence of any reflecting structures.
- Because of the nonlinearity of the feedback, with a coefficient proportional to the photon flux density of the igniting beam, excitation of nuclei is released in an avalanche-like manner, which is accompanied by emission of a giant pulse of gamma quanta.
- At present, the implementation of such a process is impeded by the absence of source of igniting gamma quanta, with a sufficient photon flux density. Therefore, the advantages of the proposed technique may manifest themselves only in designing a final stage of a source

of gamma quanta (e.g., in x-ray or gamma-ray laser, relativistic undulator, free electron laser, etc.) for production a short giant pulse of coherent gamma photons.

(I) *External ignition of two-quantum IGE of free positronium atoms.*

Antimatter as a perfect source of states with negative temperature [19, 20] attracts for a long time attention of the researches which are looking for a way to achieve a self-maintaining photon chain reaction or, in other words, annihilation laser. The main theoretical efforts have been concentrated on the least exotic annihilation reaction - the radiative electron-positron annihilation and conditions for development of gamma-ray lasing by induced annihilation of electrons and positrons have been discussed [7-14]. In this paper we consider a different mechanism for the production of coherent gamma-rays based on external ignition of avalanche-like induced annihilation of positronium atoms by conterpropagating intense photon beams.

As is well known, annihilation of free electrons and positrons with low relative velocities goes mainly through the stage of hydrogen-like bound state of electron and positron named as the positronium. There are two types of the positronium atoms in the ground state: with antiparallel spins of the electron and positron (parapositronium) and with parallel spins of the electron and positron (orthopositronium). One-photon radiative annihilation of free positronium atoms is completely forbidden. According to the lows of energy and momentum conservation annihilation of a parapositronium atom in the system of center of mass is accompanied by emission of two quanta with equal energies $\hbar\omega_0 \approx \mathrm{mc}^2 = 0.511$ keV and with opposite directed momenta. Therefore, an ensemble of parapositronium atoms is the most appropriate system for applying the method of external ignition of induced annihilation by counterpropagating photon beams.

The process of ignition of induced annihilation of positronium atoms has a number of positive and negative distinguishing features [21]. First of all, in contrast to the above nuclear case, the Doppler-free induced two-quantum emission is now impossible. Indeed, a parapositronium atom in a motion can not emit two identical quanta in opposite directions because of the complete disappearance of both emitting particles. As a result, only a small part $\varepsilon = \Delta\omega_0/\omega_D$ of parapositronium atoms ($\Delta\omega_0$ - is the transition frequency band width

equal to the inverse lifetime $1/\tau$ of parapositronium, ω_D - is the Doppler width of the emission line) turn out to be involved into the process of induced annihilation caused by counterpropagating photon beams.

Smallness of the particle masses bring to the following positive point: it is possible to lower the requirements to the igniting photon source by making use of a relativistic motion of positronium atoms to transform the photon frequency, photon density in the beam and the angular divergence of the photon beam. Due to Doppler transformation, the energy of the contrary igniting photons can be decrease down to the value

$$\hbar\omega_{ign} = \frac{mc^2}{\gamma + \left(\gamma^2 - 1\right)^{1/2}} \tag{8}$$

where γ - is the relativistic factor of the positronium beam. For example, if the energies of electrons and positrons in a beam are equal to $mc^2\gamma \approx 260$ MeV (γ =500), the energy of igniting photons becomes $\hbar\omega_{ign} = 0.5$ keV.

Simultaneously, the igniting beam photon density in the coordinate system traveling together with the positronium atoms increases by factor γ and the solid angle of the photon beam decreases by $\left[\gamma + \left(\gamma^2 - 1\right)^{1/2}\right]^2 \approx 4\gamma^2$. This means, that the brightness (spectral-angular photon flux density) of the photon beam increases $4\gamma^3$ times and for the above numerical example consist 5.10^8.

Of cause, the other beam of igniting photons, which propagate in the same direction as the positronium atoms, is subjected to inverse Doppler transformation, so the energy of photons of this beam should be extremely large $2mc^2\gamma$ =0.5GeV. Fortunately, there might be no need in the second external igniting source at all. The spontaneous photons emitted in spontaneous-stimulated annihilation transitions caused by the first igniting photon beam only are perfectly matched both in frequency and direction to play the role of the contrary igniting photon beam.

In the coordinate system moving together with the beam of parapositronium atoms with concentration N the steady-state amplification of counterpropagating beams of gamma

quanta of frequency ω_0 with photon flux densities I and I^* [$cm^{-2}s^{-1}$] within the body angle $\Delta\Omega$ and the spectral band $\Delta\omega_0$ is governed above the ignition threshold by the equations [22]

$$\frac{dI}{dz} = -\frac{dI^*}{dz} = \beta\varepsilon NII^* \tag{9}$$

where the rate constant β for two-quantum stimulated-stimulated annihilation transitions is determined by the expression

$$\beta = \frac{\lambda_C^4}{2}\frac{1}{\Delta\omega_0\Delta\Omega} \tag{10}$$

and $\lambda_C = 2\pi\hbar/mc$ - is the Compton wavelength.

A positive gain with $dI/dz > 0$ and $-dI^*/dz > 0$ is achieved under the following condition

$$I_{ign} > \frac{(1+\mu)\sigma(N_+ + N_-)/N_0}{2\varepsilon\beta}, \tag{11}$$

which can be regarded as a threshold condition for ignition and where $(N_+ + N_-)$ is the total concentration of electrons and positrons, N_0 - initial concentration of positronium atoms and σ - the cross section for photon scattering from igniting beams, $\mu = I_{ign}/I^*_{ign}$ is the ignition asymmetry coefficient.

Integration of equations (9) yields a transcendent equation for the pure output photon flux density $I_n = I_{out} - I_{ign} = I^*_{out} - I^*_{ign}$ emerging from a gain region of length L

$$\left(\frac{I_n}{I_{ign}} + 1 + \mu\right)^{-1} \ln\left[\left(\frac{I_n}{I_{ign}} + 1\right)\left(\mu\frac{I_n}{I_{ign}} + 1\right)\right] = I_{ign}\beta\varepsilon\tilde{N}L \tag{12}$$

where $\tilde{N}$ is averaged concentration of positronium atoms

$$\tilde{N} = \frac{1}{L}\int_0^L N(z)dz \tag{13}$$

Figure 2 displays the dependence of I_n/I_{ign} on the activity parameter $A = I_{ign}\beta\varepsilon\tilde{N}L$ of the gaining medium for different values of the asymmetry coefficient μ. The specific feature of these curves are the lack of one-to-one correspondence at a high enough values of the activity parameter A. As the parameter reaches its critical value $A_{cr} \sim 0.1$, the output intensity is switched to the upper branch of the S-like curve in an avalanche manner. This process is accompanied by an abrupt increase of the induced annihilation rate, which give rise to the emission of a giant pulse of coherent gamma photons.

Numerical estimates are given for the beam in which the number of parapositronium atoms 10^2 times less than the total number of electrons and positrons and a fraction of parapositronium atoms interacting with igniting photon beams consists $\varepsilon = 10^{-6}$. In accordance with (11), the threshold brightness of igniting photon beams can be estimated as $I_{ign}/(\Delta\omega_0\Delta\Omega) > 8.10^{22}$cm^{-2}steradian^{-1} and $I^*_{ign}/(\Delta\omega_0\Delta\Omega) > 8.10^{20}$ cm^{-2}steradian^{-1}, where we assumed μ=100 and took into account that the loss of gamma quanta from igniting beams mainly occur through the Compton scattering with the cross section $\sigma = 2{,}6.10^{-25}$cm^2.

The activity parameter critical for starting avalanche-like induced annihilation of parapositronium atoms is determined by the condition $A_{cr} = \left(I_{ign}\beta\varepsilon\tilde{N}L\right)_{cr} \sim 0.1$, which can be satisfied with threshold brightness $I_{ign}/(\Delta\omega_0\Delta\Omega) = 8.10^{22}$cm^{-2}steradian^{-1} for the average concentration of positronium atoms $\tilde{N} = 10^{18}$cm^{-3} and the length of gain region $L = 10$m . Relativistic motion of parapositronium atoms with $\gamma \approx 500$ leads to reduction of the threshold brightness by factor $4\gamma^3$=5.10^8 and the energy of igniting photons by factor 2γ=10^3. As a result, the threshold energy brightness of the igniting beam will be $0{,}8.10^{17}$eV cm^{-2} steradian^{-1} and becomes attainable by x-ray lasers.

The performed analysis reveals the main advantages and drawbacks of the method of external ignition of induced two-quantum annihilation of parapositronium atoms, which can be summarized as follows:

- A specific dynamic distributed feedback, which is characteristic of induced two-quantum emission in counterpropagating photon beams only, is established in the absence of any reflecting structures.
- Due to nonlinearity of the feedback, the induced annihilation of parapositronium atoms is realized in avalanche-like manner, which is accompanied by the emission of a giant pulse of coherent gamma quanta.
- It is possible to lower the requirements to the igniting photon source by making use of a relativistic motion of positronium atoms to reduce the photon energy, the photon density and the angular divergence of one of the igniting beam.
- There might be no need in the second external igniting beam at all. The spontaneous photons emitted in spontaneous-stimulated annihilation transitions caused by the first igniting photon beam only are perfectly matched both in frequency and direction to play the role of the contrary igniting photon beam.
- At present, the implementation of such a process is impeded by the absence of monokinetized parapositronium beams and by the absence of sources of igniting photons, which combine a sufficient pulse duration with high brightness. Therefore, the advantages of the proposed technique may manifest themselves only in designing a final stage of a source of gamma quanta (e.g., in x-ray or gamma-ray laser, relativistic undulator, free electron laser, etc.) for production a short giant pulse of coherent gamma photons.

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Significance

There is a threefold significance to this theoretical result:

1. The performed analysis shows the possibility of ignition by two counterpropagating photon beams a burst generation of coherent gamma radiation in an ensemble of free

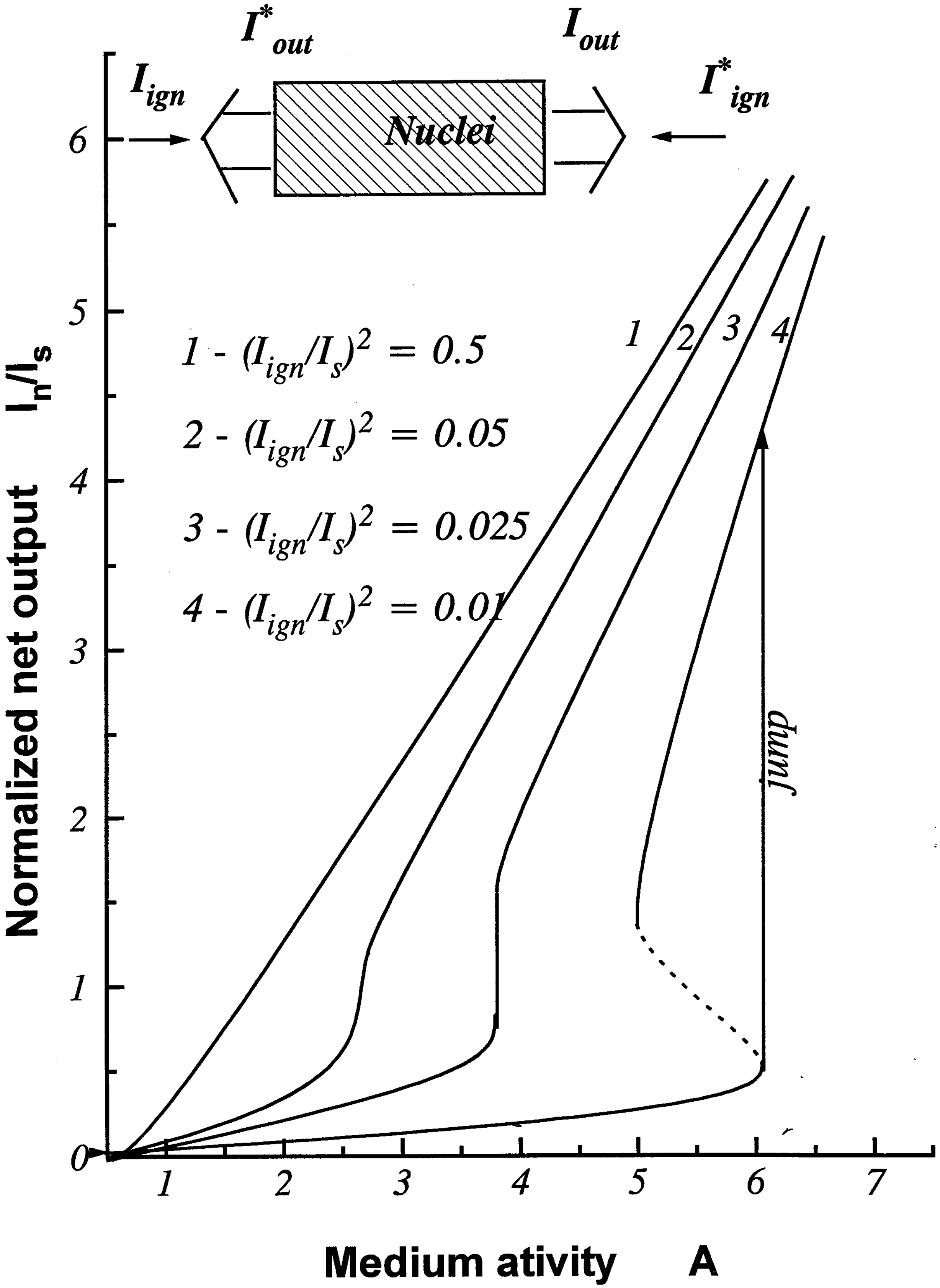
positronium atoms or in a gas of free excited nuclei without any cooling or embedding of the nuclei into a solid matrix.

2. In contrast to one-quantum emission with Doppler-broadened line, all nuclei regardless of their random individual velocities turn out to be involved into the process of avalanche-like two-quantum induced gamma emission of photons with the energies near to a half of the nuclear transition energy.

3. Such kind of a burst generation of coherent gamma photons is due to the special type of dynamics distributed feedback arising in the process of two-photon induced emission in counterpropagating beams without any reflecting structures, creating of which for the gamma-ray range is a very complicated task.

Publications

1. L.A.Rivlin, A.A.Zadernovsky. Induced Two-Quantum Gamma Emission of Free Nuclei under Conditions of External Ignition. Laser Physics, Vol. 6, No. 5, 1996, pp.956-962.
2. L.A.Rivlin, A.A.Zadernovsky. Induced Annihilation of Positronium Atoms under Conditions of External Ignition. (to be published in Laser Physics).



I^*_{out}
I_{out}
I_{ign}
I^*_{ign}
Nuclei
Normalized net output I_n/I_s
$1 - (I_{ign}/I_s)^2 = 0.5$
$2 - (I_{ign}/I_s)^2 = 0.05$
$3 - (I_{ign}/I_s)^2 = 0.025$
$4 - (I_{ign}/I_s)^2 = 0.01$
1
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3
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jump
0
1
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3
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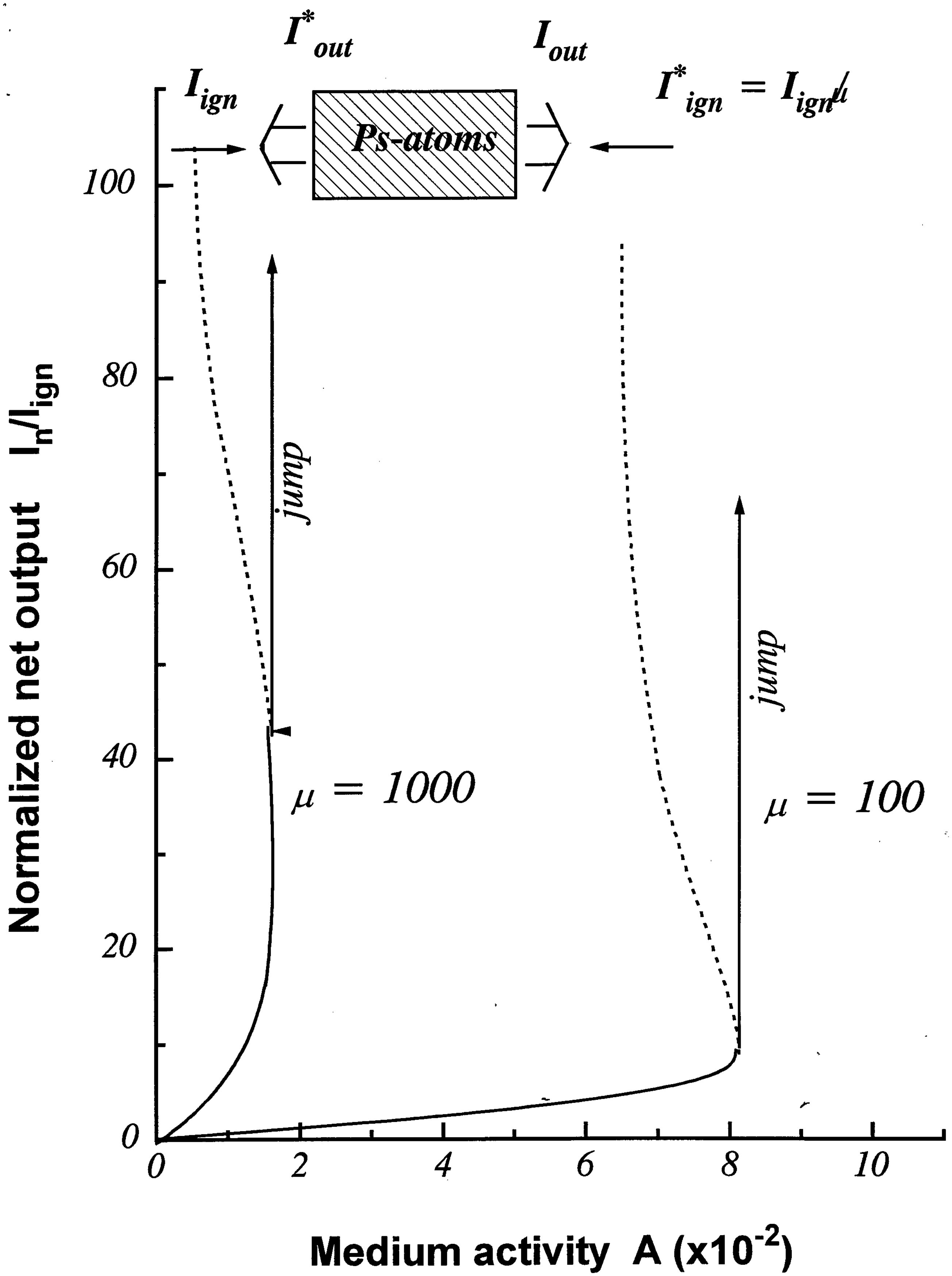
5
6
7
Medium ativity A

Figure 1



I^*_{out}
I_{out}
I_{ign}
$I^*_{ign} = I_{ign}/\mu$
Ps-atoms
jump
jump
$\mu = 1000$
$\mu = 100$
0
20
40
60
80
100
0
2
4
6
8
10
Normalized net output I_n/I_{ign}
Medium activity A (x10^{-2})

Laser Physics, Vol. 6, No. 5, 1996, pp. 956–962.

Induced Two-Quantum Gamma Emission of Free Nuclei under Conditions of External Ignition

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Abstract—The method of external ignition of stimulated two-quantum gamma emission of free excited nuclei by counterpropagating photon beams is considered. In contrast to one-quantum gamma emission of an ensemble of nuclei with a Doppler-broadened gain line, in the case under consideration, virtually all nuclei, regardless of their random individual velocities, are involved in the emission of gamma quanta into a selected mode. A specific dynamic distributed feedback, which is characteristic of stimulated two-quantum emission only and which is established without any reflective structures, is revealed. Because of the nonlinearity of the feedback, with a coefficient proportional to the intensity of the photon beam, excitation of nuclei is removed in an avalanche-like manner, which is accompanied by the emission of a giant pulse of gamma quanta.

1. INTRODUCTION

Vain attempts to design a gamma-ray laser using Mössbauer phononless transitions in nuclei embedded in a matrix of a solid, which have been undertaken during many years (e.g., see [1–4]), cause us to think of alternative approaches to this problem.

In fact, the reason for considering the possibility of using phononless nuclear transitions is associated with the fact that such transitions provide an opportunity to increase the cross section of stimulated emission by narrowing the line width of spontaneous emission to its limit through eliminating the influence of the thermal motion of atoms. In this context, a Mössbauer line with a natural radiative width can be considered as an idealized situation when the cross section of stimulated emission ceases to depend on the matrix element of transition and reaches its maximum.

It is unlikely that such an idealized situation can be implemented in practice both because of various sources of inhomogeneous broadening of nuclear lines in solids, which are difficult to eliminate in operating lasers, and because of homogeneous broadening due to the presence of other, parallel to radiative transitions, channels of relaxation of excited states (primarily through inner conversion). In addition, it is impossible to implement an ideal situation because the width of the lower laser level is finite if this level does not coincide with the ground state.

Therefore, in order to increase the cross section of stimulated emission, we should search for the methods to achieve the maximum narrowing of the spontaneous emission line rather than seek for the conditions when a phononless Mössbauer line with a natural width can be obtained. Within the framework of such an approach, there is no need to place nuclei in a matrix of a solid, which considerably complicates the technique [1–4]. This brings us to the consideration of free nuclei in gases and beams of particles [5–9].

The main source of line broadening that is to be eliminated in such a situation is a chaotic motion of nuclei (including thermal motion). Analysis of the required narrowing of a Doppler line corresponding to a one-photon transition [5–7] by ensuring a monokinetic motion of nuclei in the longitudinal direction with respect to the expected direction of the beam of gamma quanta indicates the necessity of lowering the effective longitudinal temperature of atoms or ions down to a submicrokelvin level, which seems to be not impossible to date. However, there exists an alternative method that makes it possible to eliminate an adverse effect of the chaotic motion of nuclei [8, 9] and that does not imply deep cooling. This method is based on a rich experience of sub-Doppler two-quantum spectroscopy.

2. TWO-QUANTUM STIMULATED EMISSION IN COUNTERPROPAGATING PHOTON BEAMS

As it follows from energy and momentum conservation in the emission of two photons with exactly opposite directions of wave vectors, the photon energies $\hbar\omega_1$ and $\hbar\omega_2$ are related to the energy $E_0 = \hbar\omega_0$ of the quantum transition by the expression

$$\hbar(\omega_1 + \omega_2) = E_0 + \hbar\delta\omega(u/c) - (\hbar\delta\omega)^2/(2Mc^2), \quad (1)$$

where M is the mass of the emitter, $\delta\omega = \omega_1 - \omega_2$ is the frequency detuning of counterpropagating photons, u is the projection of the emitter velocity on the direction of the wave vectors of the first photon, and c is the speed of light.

Let us assume, first, that the homogeneous line width $\Delta\omega_0$ of the laser transition is negligibly small

($\Delta\omega_0 = 0$). Then, as can be seen from (1), all the emitters with any individual velocities (a chaotic spread of these velocities causes inhomogeneous broadening) are involved in the process of emission into the selected central mode only when the frequencies of two photons resulting from the two-quantum transition exactly coincide with each other, $\omega_1 = \omega_2 = \omega_0/2$, i.e., $\delta\omega = 0$. If $\delta\omega \neq 0$, only a part of emitters having the velocity u determined by (1) contribute to the emission of a certain mode with $\omega_1 \neq \omega_2$. As is well known, these simple facts, which can be reduced to the compensation for the first-order Doppler frequency shift for counterpropagating beams of photons with equal frequencies, provide the basis for the method of sub-Doppler spectroscopy.

Now, if we take into account that the homogeneous line width of the transition $\Delta\omega_0$ is finite, then we find from (1) that emission involving virtually all emitters with arbitrary velocities is possible not only for the central mode with $\omega_1 = \omega_2 = \omega_0/2$ and $\delta\omega = 0$ but also for a group of modes with $\delta\omega \neq 0$. Each mode from this group has a homogeneous line width $\Delta\omega_0$. The admissible frequency detuning for this group of modes is determined by the requirement that the difference of sum frequencies of emitted photons should not exceed the homogeneous line width of the transition,

$$(\omega_1 + \omega_2)_{\max} - (\omega_1 + \omega_2)_{\min} \leq \Delta\omega_0, \qquad (2)$$

where the extremal values of frequencies correspond to the maximum, $u_{\max}$, and minimum, $u_{\min}$, values of the velocity distribution of emitters with a variance $\Delta u = u_{\max} - u_{\min}$. Then, in accordance with (1), the admissible detuning is given by

$$|\delta\omega_0| = \Delta\omega_0(c/\Delta u), \qquad (3)$$

and the number of modes in such a group is

$$N = |\delta\omega_0|/\Delta\omega_0 = c/\Delta u \gg 1. \qquad (4)$$

In a certain sense, the detuning $\delta\omega_0$ in (3) can be considered as a specific type of inhomogeneous broadening that does not exclude the overwhelming majority of emitters from the interaction with the field of each mode from this group.

Thus, when we irradiate an inverted ensemble of nuclei with two counterpropagating igniting photon beams produced by an external source, the spectrum of stimulated emission displays a maximum with a width on the order of $|\delta\omega_0|$ (3) around the central frequency $\omega_1 = \omega_2 = \omega_0/2$ (with an accuracy up to a small shift $-\hbar(\delta\omega_0)^2/(2Mc^2)$, which is due to the recoil effect). This maximum consists of N modes (4) with a homogeneous width $\Delta\omega_0$ each and represents the contribution of virtually all nuclei with all possible velocities of chaotic longitudinal motion. Beyond the limits of the frequency range determined by (3), this maximum is observed against a lower intensity background associated with emission of separate groups of nuclei that belong to different parts of the nuclear velocity distribution.

Thus, the above-described approach eliminates an adverse effect of a nonmonokinetic character of an ensemble of free nuclei without a deep cooling of these nuclei. Simultaneously, as will be demonstrated below, this approach makes it possible to establish an effective feedback within the gamma range, where the creation of mirrors or some other reflective structures encounters considerable difficulties.

3. AMPLIFICATION OF COUNTERPROPAGATING BEAMS OF GAMMA QUANTA

The steady-state amplification of counterpropagating beams of gamma quanta that belong to one of N modes (4) in the considered group of modes with flux densities I and I^* [$\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$] within the spectral band $\Delta\omega_0$ is governed by the equations

$$dI/dz = \beta(n_2 - n_1)II^* + \gamma n_2(I + \mu I^*) + \mu_0\alpha n_2 - \sigma n I, \qquad (5)$$

$$-dI^*/dz = \beta(n_2 - n_1)II^* + \gamma n_2(I^* + \mu I) + \mu_0\alpha n_2 - \sigma n I^*, \qquad (6)$$

where n_2 and n_1 are the concentrations of nuclei in the upper and lower levels of the laser transition, respectively, and n is the total concentration of nuclei. The first terms in these equations describe stimulated two-quantum emission with a coefficient β [cm^4 s]. The second terms take into account spontaneous–stimulated emission of the beams with flux densities I and I^* into the considered modes with a coefficient γ [cm^2]. The third terms describe purely spontaneous emission into the same modes with a coefficient α [s^{-1}]. The last term characterizes the total loss of photons from the mode with a scattering cross section σ [cm^2]. The factors $\mu = \Delta\Omega/4\pi$ and $\mu_0 = \mu\Delta\omega_0/\omega_0$ specify the fractions of photons emitted into a solid angle $\Delta\Omega$ that covers the modes of the beams I and I^* and into the band with a homogeneous width $\Delta\omega_0$, respectively. The longitudinal coordinate z is chosen in such a manner that $z = 0$ coincides with the center of the gain region of length L.

A positive gain with $dI/dz > 0$ and $-dI^*/dz > 0$ is achieved if

$$\beta(n_2 - n_1)II^* > (1/2)[\sigma n - \gamma n_2(1 + \mu)](I + I^*) - \mu_0\alpha n_2. \qquad (7)$$

Hence, if the intensities of the igniting photon beams satisfy the equality $I = I^* = I_i$ at the input of the gain region, we find the following threshold condition of ignition:

$$I_i > I_0 \equiv \frac{\sigma n - \gamma n_{20}(1 + \mu)}{\beta n_0}. \qquad (8)$$

Since the factor μ_0 is small, we omitted the last term in (7), which is responsible for spontaneous emission.

The quantity $n_0 = n_{20} - n_{10}$ in (8) stands for the initial value of the inverted population, and n_{20} is the concentration of excited nuclei in the upper level of the laser transition in the absence of an igniting photon beam.

If the beam intensities I and I^* are much higher than the threshold level determined by (8), so that $II^* \gg I_0^2$, inequality (7) becomes so strong that we can keep only the first terms in (5) and (6). Then, we have $d(I + I^*)/dz = 0$ and $I + I^* = I_i + I_L = \text{const}$, where I_L is the intensity of the beams I and I^* at the output of the gain region, i.e., at $z = L/2$ and $z = -L/2$, respectively. Correspondingly, equation (5) is reduced to

$$dI/dz = \beta(n_2 - n_1)I(I_i + I_L - I), \qquad (9)$$

and the steady-state inverted population difference for the laser levels is given by

$$n_2 - n_1 = \frac{n_0}{1 + II^*/I_s^2}, \qquad (10)$$

where the saturation parameter I_s depends on the specific configuration of levels and the method of pumping.

4. DYNAMICS OF AMPLIFICATION IN A TWO-QUANTUM PROCESS

Integration of equation (9) with allowance for the saturation of excessive population (10) yields a transcendental equation for the pure output intensity $I_N = I_L - I_i$ emerging from a gain region of length L,

$$2\left(\frac{I_N}{I_s} + 2\frac{I_i}{I_s}\right)^{-1} \ln\left(\frac{I_N}{I_i} + 1\right) + \frac{I_N}{I_s} = \beta n_0 I_s L. \qquad (11)$$

Figure 1 displays the dependence of I_N/I_s on the product $\beta n_0 I_s L$ found from this equation, with the squared ratio $(I_i/I_s)^2$ taken as a parameter of a family of curves.

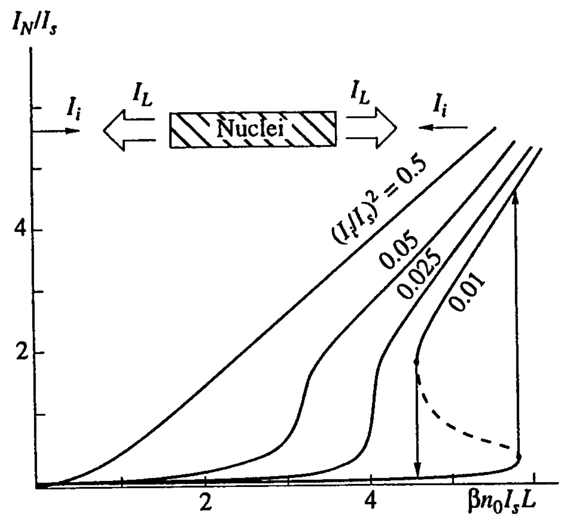

Figure.

The specific features of these curves are the lack of one-to-one correspondence and a hysteresis character. For a sufficiently high ignition intensity, $(I_i/I_s)^2 > 0.025$, the output intensity I_N/I_s displays a smooth increase with a growth in $\beta n_0 I_s L$. Note that the growth rate of I_N/I_s becomes greater with an increase in I_i/I_s.

An ambiguity in the considered curves that occurs for a low intensity of the igniting photon beam, $(I_i/I_s)^2 < 0.025$, decreases the growth rate of I_N/I_s in the initial section of the dependence. As the argument $\beta n_0 I_s L$ reaches its critical value, the output intensity is switched to the upper branch of the S-like curve in an avalanche manner. This process is accompanied by an abrupt devastation of the population (obviously, steady-state solutions do not describe this effect), which gives rise to the emission of a pulse of gamma photons.

When $\beta n_0 I_s L$ decreases and approaches unstable sections of the curves, shown by dashed lines in Fig. 1, an evolution of I_N/I_s may be accompanied by a hysteresis with a jump downward.

The critical value of the product $(\beta n_0 I_s L)_{cr}$ that corresponds to an avalanche-like jump is determined by the condition $d(I_N/I_s)/d(\beta n_0 I_s L) = \infty$. With allowance for (11), this condition yields

$$\frac{(\beta n_0 I_s L)_{cr}}{2} = \frac{1}{(I_N/I_s)_{cr} + I_i/I_s} + \left(\frac{I_N}{I_s}\right)_{cr} + \frac{I_i}{I_s}. \qquad (12)$$

As can be seen from this relationship, there are no critical points for $\beta n_0 I_s L < 4$ at any value of the parameter I_i/I_s. Provided that $\beta n_0 I_s L > 4$, critical points occur for sufficiently small I_i/I_s. Within the most interesting range of large avalanche-like jumps, where $(\beta n_0 I_s L)_{cr} \gg 4$, we can estimate the required intensity of igniting pulses assuming that $(I_N/I_s)_{cr} \approx 0$ at the lower starting point of the jump,

$$I_i/I_s = 2/(\beta n_0 I_s L)_{cr}. \qquad (13)$$

Discontinuities and hysteresis in dependences shown in Fig. 1 are due to the dynamic distributed feedback, which arises in induced two-quantum emission in counterpropagating photon beams.

5. DYNAMIC DISTRIBUTED FEEDBACK

One of the most complex problems that arise when we analyze the possibility of creating a gamma-ray laser is associated with the necessity to implement positive feedback for electromagnetic radiation where the energy of photons exceeds tens of kiloelectronvolts. Proposals to use Bragg reflection in single crystals for creating mirrors and establishing distributed feedback [1–3, 10] have not received acceptance thus far.

Within the framework of the approach under consideration, one can also solve the problem of distributed feedback because such a feedback is inherent in stimulated two-quantum emission in the field of two counterpropagating beams of photons with equal frequencies [11].

Such counterpropagating beams produce a standing wave in the amplification region (the main attribute of a feedback) in the absence of any mirrors or periodic scattering structures. Formation of such a standing wave does not require any material dynamic grating.

Indeed, conventional distributed feedback based on a stationary periodic scattering structure [11] is characterized by a nonzero coefficient of coupling ρ of counterpropagating waves. This coupling coefficient is defined as a variation in the photon flux density dI^* in the backward wave within length element dz divided by the flux density I in the forward wave,

$$\rho dz = \frac{1}{I}\frac{dI^*}{dz}dz. \quad (14)$$

In an induced two-quantum process in the field of counterpropagating waves, the photon flux density in the backward wave changes in each event of stimulated emission. Because of intrinsic features of induced emission, newly created photons are perfectly phase-matched and are emitted into an appropriate mode. Therefore, according to (9), the nonlinear coefficient of dynamic distributed feedback in induced two-photon emission is given by

$$\rho = \beta(n_2 - n_1)I^* \quad (15)$$

and increases proportionally to I^*.

6. TRANSITION PROBABILITIES AND LINE BROADENING

The probability of a spontaneous–spontaneous two-quantum transition accompanied by emission of one of the photons within the spectral interval $d\omega_1$ near the frequency ω_1 per unit time is

$$dW_{ss} = W_{ss}f(\omega_1)d\omega_1, \quad (16)$$

where $W_{ss} = \tau_{2\gamma}^{-1}$ is the inverse lifetime of a nucleus in the upper level with respect to a two-quantum radiative transition. The line contour of the frequency distribution of emitted photons $f(\omega_1)$ normalized to unity is written in terms of an integral over all admissible frequencies ω_2 of the second quantum and over all emission directions of both quanta,

$$f(\omega_1) = \frac{\tau_{2\gamma}}{(2\pi)^3}\int\left(\sum_{\sigma_1\sigma_2}|M_{12}|^2\right)\left(\frac{\omega_1}{c}\right)^3\left(\frac{\omega_2}{c}\right)^3$$
$$\times g(\omega_0 - \omega_1 - \omega_2)d\Omega_1 d\Omega_2 d\omega_2 \quad (17)$$
$$= \frac{\tau_{2\gamma}}{(2\pi)^3}\left(\frac{\omega_1}{c}\right)^3\left(\frac{\omega_0 - \omega_1}{c}\right)^3\int\left(\sum_{\sigma_1\sigma_2}|M_{12}|^2\right)\Bigg|_{\omega_2 = \omega_0 - \omega_1} d\Omega_1 d\Omega_2.$$

The Lorentz function $g(\omega_0 - \omega_1 - \omega_2)$ in (17) with a frequency bandwidth $\Delta\omega_0$, which is equal to the sum of the widths of the upper and lower levels, takes into account a resonant character of transition. For a pair of electrically dipole quanta ($E1$, $E1$) with frequencies ω_1 and ω_2 and polarization vectors $\mathbf{e}_{\sigma_1}$ and $\mathbf{e}_{\sigma_2}$ ($\sigma_1, \sigma_2 = 1, 2$), the matrix element M_{12} is given by the sum over intermediate nuclear states with energies E_n,

$$M_{12} = \sum_n\left(\frac{\langle 1|\mathbf{e}_{\sigma_2}\mathbf{d}|n\rangle\langle n|\mathbf{e}_{\sigma_1}\mathbf{d}|2\rangle}{E_2 - E_n - \hbar\omega_1}\right.$$
$$\left.+\frac{\langle 1|\mathbf{e}_{\sigma_1}\mathbf{d}|n\rangle\langle n|\mathbf{e}_{\sigma_2}\mathbf{d}|2\rangle}{E_2 - E_n - \hbar\omega_2}\right). \quad (18)$$

If the neighboring intermediate level lies between the levels involved in the considered $2 \longrightarrow 1$ transition and has an energy $E_s = \hbar\omega_s$ measured relative to the energy of the lower level 1, and the energy separation of this level from the center of the energy interval corresponding to the transition under consideration, $|E_s - E_0/2|$, is much greater than the width of this level, then the matrix element (18) is a nonresonant slowly varying function within the frequency band centered at $\omega_1 = \omega_0/2$ whose spectral width is much less than $|\omega_s - \omega_0/2|$. Using for these frequencies the estimate

$$\int\left(\sum_{\sigma_1\sigma_2}|M_{12}|^2\right)\Bigg|_{\omega_2 = \omega_0 - \omega_1} d\Omega_1 d\Omega_2$$
$$\approx\left(\frac{32\pi}{3}\right)^2\frac{1}{E_0^2}\frac{(ea)^4}{(1 - 2E_s/E_0)^2}, \quad (19)$$

we derive

$$dW_{ss} = (2\pi)^3\left(\frac{16}{3}\right)^2\left(\frac{\omega_1}{\omega_0}\right)^3\left(1 - \frac{\omega_1}{\omega_0}\right)^3$$
$$\times\frac{\alpha_0^2(a/\lambda_0)^4}{(1 - 2E_s/E_0)^2}d\omega_1, \quad (20)$$

where λ_0 is the wavelength of radiation with the energy of quanta equal to the energy E_0 of transition under consideration, $\alpha_0 = e^2/(\hbar c) = 1/137$ is the fine-structure constant, and $a = 1.3 \times 10^{-13}A^{1/3}$ cm is the radius of a nucleus where the number of nucleons is equal to A. Correspondingly, the coefficient α, which is involved in equations (5) and (6) and which describes the contribution of spontaneous emission to the modes of counterpropagating beams with frequencies close to half the transition frequency, is given by

$$\alpha = (2\pi)^3\frac{4}{9}\frac{\alpha_0^2(a/\lambda_0)^4}{(1 - 2E_s/E_0)^2}\omega_0. \quad (21)$$

It is convenient to define the probability of stimulated–spontaneous transitions per unit time with the use of the Einstein relationship $A(\omega_1)/B(\omega_1) = \hbar\omega_1^3/(\pi^2 c^3)$ for spectral coefficients of spontaneous emission, $A(\omega_1)$,

and stimulated emission, $B(\omega_1)$. Indeed, according to (16), the spectral coefficient $A(\omega_1)$ of spontaneous emission is equal to $W_{ss}f(\omega_1)$. Then, using the Einstein relationship, we can derive an expression for the probability of stimulated–spontaneous transitions with induced emission of photons within the spectral interval $d\omega_1$ near the frequency ω_1 (corresponding to the wavelength λ_1) per unit time,

$$dW_{is} = (\lambda_1^2/4)W_{ss}f(\omega_1)I(\omega_1)d\omega_1, \qquad (22)$$

where $I(\omega_1)d\omega_1$ is the flux density of stimulating photons within the considered spectral band. Hence, with the use of estimate (20), we can find an expression for the constant γ, which is involved in equations (5) and (6) and which includes the contribution of stimulated–spontaneous transitions to the modes of counterpropagating beams with frequencies close to half the transition frequency ($\lambda_1 \approx 2\lambda_0$),

$$\gamma = (2\pi)^3\frac{4}{9}\frac{\alpha_0^2(a/\lambda_0)^4}{(1-2E_s/E_0)^2}\lambda_0^2. \qquad (23)$$

The probability of stimulated–stimulated transitions with induced emission of photons within spectral intervals $d\omega_1$ and $d\omega_2$ near the frequencies ω_1 and ω_2 per unit time is given by

$$\begin{aligned} dW_{ii} &= (\lambda_1^2/4)(\lambda_2^2/4)W_{ss}f(\omega_1) \\ &\times g(\omega_0-\omega_1-\omega_2)I(\omega_1)I^*(\omega_2)d\omega_1 d\omega_2, \end{aligned} \qquad (24)$$

where $I(\omega_1)d\omega_1$ and $I^*(\omega_2)d\omega_2$ are the flux densities of stimulating photons. Correspondingly, the constant β of stimulated two-quantum emission, which is involved in equations (5) and (6), is written as ($\lambda_1 = \lambda_2 = 2\lambda_0$)

$$\beta = \left(\frac{8\pi}{3}\right)^2\frac{\alpha_0^2 a^4/\Delta\omega_0}{(1-2E_s/E_0)^2}. \qquad (25)$$

To take into account the motion of emitters, we should include the Doppler shift of the frequencies ω_1 and ω_2 in the right-hand side of (17). In particular, the Lorentz functions should be replaced by the distribution function

$$g(\omega_0 - \hbar(\delta\omega)^2/(2Mc^2) + \delta\omega(u/c) - \omega_1 - \omega_2), \qquad (26)$$

which gives the relation between the frequencies of counterpropagating quanta emitted by a nucleus with the velocity projection on the direction of emission of the first quantum equal to u. Next, we should multiply the derived expression by the probability that the velocity projection of a nucleus falls within the interval from u to $u + du$,

$$F(u)du = \left(\frac{M}{2\pi kT}\right)^{1/2}\exp\left(-\frac{Mu^2}{2kT}\right)du, \qquad (27)$$

where M is the mass of the nucleus and T is the temperature of the gas, and integrate over all possible velocity projections of the nucleus. This procedure yields a Doppler-broadened line corresponding to the frequency distribution of emitted photons.

However, we should note that, if the frequency detuning $\delta\omega$ between counterpropagating photon beams is not very large, so that

$$|\delta\omega| \ll |\delta\omega_0| = \Delta\omega_0 c/\Delta u = \Delta\omega_0(Mc^2/kT)^{1/2}, \qquad (28)$$

where $\Delta u = (kT/M)^{1/2}$ is the variance of the nuclear velocity corresponding to the distribution function (27), we can neglect the Doppler term $\delta\omega u/c$ in the argument of the Lorentz function (26) for the overwhelming majority of the emitters.

Provided that, in addition, the term $\hbar(\delta\omega)^2/(2Mc^2)$ in the argument of the function (26), which describes the recoil effect in emission, is much less than the homogeneous width $\Delta\omega_0$ of the transition, which is true for the detunings that satisfy the inequality

$$|\delta\omega| \ll \Delta\omega_0(2Mc^2/\hbar\Delta\omega_0)^{1/2}, \qquad (29)$$

we can neglect this term as well.

Comparison of inequalities (28) and (29) shows that, if $2kT > \hbar\Delta\omega_0$, which is usually true for nuclear transitions, the Lorentz function (26) for a moving nucleus can be replaced by the Lorentz function for a motionless nucleus. Then, the integration over all possible velocity projections u is reduced to the integration of $F(u)$ defined by (27), which yields unity. This implies that stimulated emission into a group of modes within the band (28) near the central frequency $\omega_0/2$ involves virtually all nuclei in the gas. As a result, the spectrum of stimulated emission features a maximum with a width on the order of $|\delta\omega_0| = \Delta\omega_0 c/\Delta u$ near the central frequency $\omega_0/2$.

Note also that, for the overwhelming majority of nuclei in a gas, the Doppler shift of frequencies of emitted quanta should not exceed the frequency separation from the neighboring intermediate level. Otherwise, in analyzing the motion of nuclei, we should take into account resonant denominators in the matrix element M_{12} (18), and the estimate (20) and, consequently, formulas (21), (23), and (25) become inapplicable. In addition, what is even more important, in the case of an exact resonance, stimulated two-quantum transitions do not offer the above-considered advantages any longer because such transitions occur through two sequential cascade single-quantum transitions $2 \longrightarrow s$ and $s \longrightarrow 1$. Thus, although it is desirable to ensure a small denominator in the expression for the constant β of stimulated two-quantum emission (25), the detuning from the exact resonance should be limited by a value that is considerably greater than half the Doppler width

of the line corresponding to a single-quantum cascade transition,

$$(1-2E_s/E_0)^2 \gg 2(kT/Mc^2)\ln 2 \approx 1.2\times 10^{-13}(T/A). \quad (30)$$

To be able to neglect the components of homogeneous broadening due to a finite transit time and collisions, we should require that the inverse transit time $(\Delta t)^{-1}$ of nuclei through the amplification region and the inverse time interval between collisions should be small, $(\Delta t)^{-1} \ll \Delta\omega_0$. These requirements impose restrictions on the size of the amplification region and the total concentration of nuclei n,

$$L \gg u/\Delta\omega_0 = (c/\Delta\omega_0)(3kT/Mc^2)^{1/2}, \quad (31)$$

$$n \ll \Delta\omega_0/(\sigma_g u) = (\Delta\omega_0/c\sigma_g)(Mc^2/3kT)^{1/2}, \quad (32)$$

where $u = (3kT/M)^{1/2}$ is the root-mean-square velocity of the thermal motion of nuclei and σ_g is the gas-kinetic cross section.

Finally, the second-order Doppler broadening, which is inevitable in a two-quantum process, should be less than the homogeneous width,

$$\Delta\omega_D^{(2)} \approx \omega_0(kT/Mc^2) \ll \Delta\omega_0. \quad (33)$$

Along with the thermal motion of nuclei, Doppler broadening may be due to the scatter in velocities acquired by nuclei in the process of laser pumping if the time is too short for the thermalization of nuclei to occur. Specifically, being isotropically pumped with an incoherent source of gamma photons with an energy $\hbar\omega_p$, a nucleus acquires a momentum $\hbar\omega_p/c$ of an absorbed quantum. The maximum possible velocity difference is equal to twice the acquired velocity, $2c(\hbar\omega_p/Mc^2)$, which gives an estimate for the corresponding effective temperature:

$$kT_{\text{eff}} \approx (\hbar\omega_p)^2/(Mc^2). \quad (34)$$

7. NUMERICAL ESTIMATES

In this section, we present estimates for a hypothetical "fortunate" nucleus with $A = 150$ and an intermediate level s lying between the levels involved in the laser transition with a detuning $|1 - 2E_s/E_0| = 2\times 10^{-6}$. Then, at $T = 300$ K, inequality (30) would be satisfied even if the right-hand side of this equality were larger by a factor of ten. For such a detuning, expressions (23) and (25) give the following estimate for the constant of stimulated emission: $\beta = (2.3\times 10^{-40}\ \text{cm}^4)/\Delta\omega_0$. For $E_0 = 10^5$ eV, the constant of spontaneous–stimulated emission is estimated as $\gamma = 2.4\times 10^{-22}$ cm^2.

In accordance with (8), the threshold spectral flux density of igniting photons can be estimated as $I_0/\Delta\omega_0 = 3\times 10^{18}$ cm^{-2}, where we assumed that $n_{20} = 0.8n$ and $n_0 = 0.6n$ and took into account that the scattering of gamma quanta with an energy of about 50 keV mainly occurs through photoionization of atoms with the scattering cross section $\sigma = 6\times 10^{-22}$ cm^2.

With allowance for (13), the requirement $I_i > I_0$ (8) yields $(n_0L)_{cr} < 3\times 10^{21}$ cm^{-2}. This inequality can be satisfied, for example, with $n_0 = 10^{17}$ cm^{-3} and $L = 200$ m. For $\sigma_g = 10^{-16}$ cm^2, restrictions (31) and (32) give $L \gg (2.2\times 10^4\ \text{cm s}^{-1})/\Delta\omega_0$ and $n \ll (4.5\times 10^{11}\ \text{cm}^{-3}\ \text{s})\Delta\omega_0$. These inequalities do not contradict the chosen values of the inverted population n_0 (the concentration of nuclei in this case is $n = n_0/0.6 = 1.7\times 10^{17}$ cm^{-3}) and the size L of the amplification region if the emission bandwidth meets the condition $\Delta\omega_0 > 10^8$ s^{-1}. The restriction (33) on the magnitude of the second-order Doppler effect is reduced to the inequality $\Delta\omega_D^{(2)} = 2\times 10^7\ \text{s}^{-1} \ll \Delta\omega_0$. An estimate of the effective heating by pumping in accordance with (34) yields $T = 10$ K, which is much lower than the temperature of the medium, $T = 300$ K.

According to (13), the critical spectral density of the photon flux in the igniting beams is estimated as $I_i/\Delta\omega_0 = 4.3\times 10^{18}$ cm^{-2}.

Finally, let us compare the estimated spectral density of the photon flux in the igniting beams with the capabilities of the available sources of gamma radiation. The spectral density of the photon flux in synchrotron radiation (within a solid angle of 10^{-5} sr) is estimated as approximately 10 cm^{-2} [12], which is many orders of magnitude lower than the required value of $I_i/\Delta\omega_0$. Although X-ray lasers ensure a higher spectral density of the photon flux, about 10^{15} cm^{-2} [12], such a spectral density of the photon flux is lower than the required one by three orders of magnitude. In addition, the pulse duration of radiation produced by X-ray lasers is not sufficient to ensure ignition.

8. STIMULATION OF GAMMA EMISSION FROM NUCLEI AS A METHOD OF ECOLOGICALLY SAFE POWER PRODUCTION

The horizons of using stimulated radiative processes in nuclei for power production were discussed already in the pioneering proposals on gamma-ray lasers. Specifically, creating a nuclear reactor that can be used as a source of energy both in the pulse and continuous regimes was indicated as one of the main applications of the device proposed in [13].

In fact, operation of a gamma-ray laser is one of the modifications of nuclear reactions, namely, a chain reaction of induced transitions in excited nuclei. This circumstance was highlighted in the title of the first technical report on this problem (see reference [5] in [14]).

The energy accumulated by excited states of isomeric nuclei is about 50 MJ/g (see table), which is

Table

Isomer	$_{41}Nb^{93m}$	$_{47}Ag^{108m}$	$_{67}Ho^{166m}$	$_{72}Hf^{178m}$	$_{73}Ta^{180m}$	$_{75}Re^{186m}$	$_{77}Ir^{192m}$	$_{83}Bi^{210m}$
Lifetime, years	13.6	127	1.2×10^3	31	1.2×10^{15}	2×10^5	241	3×10^6
Transition energy, keV	30	79			75			262
Spin of the initial state	$1/2^-$	6	7		9^-			9^-
Spin of the final state	$9/2^+$	1	2^-	0^+	1^+			1^-
Multipolarity	M4	M5	E5		M8			E8
Energy content, MJ/g	31	70			40			120
Time of transformation of a product into a stable isotope	0	2.4 min	27 h	0	8.1 h	91 h	74 days	138 days

approximately two orders of magnitude lower than the specific energy content of nuclear fuel used in nuclear fission and three orders of magnitude higher than the heat-producing capability of hydrocarbon chemical fuel. Advantages and drawbacks of such an intermediate position of energy released in gamma emission determine the status of possible gamma-laser chain reaction in the hierarchy of power production. The main argument in favor of such a method of power production is its ecological safety, i.e., the absence of medium- and long-lived radionuclides in the products of this reaction (see table).

Being implemented in the pulse–periodic regime, the above-considered method of the external ignition of a stimulated radiative nuclear reaction holds much promise as one of the ways to solve the problem of ecologically safe power production.

9. CONCLUSION

The performed analysis reveals the main advantages and drawbacks of the method of external ignition of stimulated two-quantum emission from free excited nuclei using counterpropagating photon beams. The advantages and drawbacks of this technique can be summarized as follows:

(1) In contrast to single-quantum emission in an ensemble of nuclei with a Doppler-broadened gain line, emission of gamma quanta into a selected mode involves virtually all nuclei regardless of their individual velocities;

(2) A specific dynamic distributed feedback, which is characteristic of stimulated two-quantum emission in counterpropagating beams only, is established in the absence of any reflective structures;

(3) Because of the nonlinearity of the feedback, with a coefficient proportional to the intensity of the photon beam, excitation of nuclei is removed in an avalanche-like manner, which is accompanied by the emission of a giant pulse of gamma quanta;

(4) At present, the implementation of such a process is impeded by the absence of sources of igniting gamma quanta with a sufficient intensity. Therefore, the advantages of the proposed technique may manifest themselves only in designing a final stage of a source of gamma quanta (e.g., in an X-ray or gamma-ray laser, relativistic undulator, free-electron laser, etc.) for producing a short pulse of gamma photons with a high peak amplitude.

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Induced Annihilation of Positronium Atoms under Conditions of External Ignition

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Abstract

Induced annihilation of positronium atoms ignited by two counterproragating intense photon beams is discussed. The performed analysis reveals the main advantages and drawbacks of the method of external ignition of induced annihilation of positronium atoms, which can be summarized as follows:

- A specific dynamic distributed feedback, which is characteristic of induced two-quantum emission in counterpropagating photon beams only, is established in the absence of any reflecting structures.
- Due to nonlinearity of the feedback, the induced annihilation of positronium atoms is realized in avalanche-like manner, which is accompanied by the emission of a giant pulse of coherent gamma quanta.
- It is possible to lower the requirements to the igniting photon source by making use of a relativistic motion of positronium atoms to reduce the photon energy, the photon density and the angular divergence of one of the igniting beam.
- There might be no need in the second external igniting beam at all. The spontaneous photons emitted in spontaneous-stimulated annihilation transitions caused by the first igniting photon beam only are perfectly matched both in frequency and direction to play the role of the contrary igniting photon beam.
- At present, the implementation of such a process is impeded by the absence of monokinetized positronium beams and by the absence of sources of igniting photons, which combine a sufficient pulse duration with high brightness.

Индуцированная аннигиляция атомов позитрония при внешнем поджиге

А.А.Задерновский, Л.А.Ривлин

1. Введение

Антиматерия, как идеальный источник состояний с отрицательной температурой [1, 2], давно привлекает внимание исследователей, ищущих пути к получению когерентной генерации гамма-квантов и, в конечном итоге, созданию гамма лазера. Особенно часто, как наименее экзотическая, рассматривалась в этом отношении реакция аннигиляции электронов и позитронов [3-10]. В данной работе исследуется возможность внешнего поджига встречными фотонными пучками лавинообразной индуцированной аннигиляции атомов позитрония.

Как известно [11], при малых относительных скоростях свободных электронов и позитронов $v<\alpha c$ (α - постоянная тонкой структуры, c - скорость света в вакууме) становится существенным кулоновское притяжение между частицами и их аннигиляция происходит в большинстве случаев через стадию образования водородоподобного связанного состояния электрона и позитрона - атома позитрония. В низшем энергетическом состоянии атом позитрония существует в двух видах: с антипараллельными спинами электрона и позитрона (парапозитроний) и параллельными спинами электрона и позитрона (ортопозитроний). Основное состояние ортопозотрония со спином равным единице трехкратно вырождено по проекциям спина и поэтому атомов ортопозитрония образуется в три раза больше, чем атомов парапозитрония. В силу законов сохранения энергии и импульса и закона сохранения зарядовой четности при электромагнитных взаимодействиях, атом парапозитрония аннигилирует с испусканием только четного числа (двух и более) фотонов, а атом ортопозитрония - с испусканием только нечетного числа (трех и более) фотонов. В связи с этим, оба вида атомов позитрония, оказываются чрезвычайно привлекательными для применения метода внешнего поджига индуцированной аннигиляции с помощью встречных интенсивных фотонных пучков.

2. Метод внешнего поджига индуцированной аннигиляции позитрония

Метод внешнего поджига встречными фотонными пучками двухквантового процесса стимулированного испускания впервые предложен в [12]. Возможность применения этого метода к колективу свободных возбужденных ядер подробно рассмотрена в [13]. Показано, что в отличие от одноквантового процесса испускания с доплеровским уширением линии усиления, в процесс индуцированного излучения гамма квантов во встречных пучках фотонов с энергиями близкими к половине энергии ядерного перехода вовлекаются практически все ядра, независимо от их случайных индивидуальных скоростей. При этом, устанавливается присущий лишь двухквантовому стимулированному испусканию специальный вид динамической распределенной обратной связи без каких-либо отражающих структур. Нелинейность обратной связи вызывает лавинообразное снятие возбуждения ядер, сопровождающееся излучением гигантского импульса гамма квантов.

Применение метода внешнего поджига к ансамблю атомов позитрония имеет ряд особенностей [14] , обусловленных аннигиляцией позитрония в процессе излучения, то есть исчезновением носителя импульса при испускании гамма квантов. В связи с этим, например, аннигиляция покоящегося атома парапозитрония сопровождается испусканием двух квантов строго противоположного направления и одинаковой энергии $\hbar\omega_0 \approx \mathrm{mc}^2 = 0{,}511$ Мэв , равной половине энергии основного состояния. Движущийся же с некоторой скоростью v атом парапозитрония не может (в отличие от свободного ядра) испустить два одинаковых гамма кванта в противоположных направлениях. Аннигиляция парапозитрония препятствует выполнению закона сохранения импульса в этом процессе. Поэтому линия аннигиляционного излучения коллектива атомов парапозитрония имеет доплеровскую ширину $\Delta\omega_{\mathrm{D}}$ и при облучении встречными поджигающими пучками фотонов, сосредоточенных в частотной полосе $\Delta\omega$ вблизи частоты ω_0 , в процесс индуцированной аннигиляции оказываются вовлеченными лишь малая доля $\varepsilon = \Delta\omega/\Delta\omega_{\mathrm{D}}$ атомов позитрония, принадлежащих центральному участку их скоростного распределения вблизи v=0 .

Возникающие в связи с этим повышенные требования к монокинетичности пучка атомов позитрония могут быть удовлетворены с помощью различных методов предварительной монокинетизации электронного и позитронного пучков, при смешивании которых образуется позитроний. Простейшим из них является метод ускорения заряженных частиц при одновременном воздействии импульсного электрического поля на все частицы, содержащиеся в межэлектродном промежутке [15].

Важным преимуществом атомов позитрония перед ядрами является возможность использования релятивистских позитрониевых пучков, что существенно снижает требования к источнику поджигающих фотонов встречного направления. Так, энергия поджигающих фотонов, которая в системе покоя парапозитрония должна быть равной $\hbar\omega_0 \approx \mathrm{mc}^2 = 0{,}511$ Мэв может быть уменьшена благодаря доплеровской трансформации до величины $\hbar\omega_{\mathrm{ign}}$, определяемой равенством

$$\hbar\omega_{\mathrm{ign}} = \frac{\mathrm{mc}^2}{\gamma + (\gamma^2 - 1)^{1/2}} \tag{1}$$

где γ - релятивистский фактор пучка атомов позитрония. Например, при энергии электронов и позитронов в пучке $\mathrm{mc}^2\gamma \approx 260$ Мэв ($\gamma \approx 500$) энергия поджигающих фотонов может быть уменьшена в 10^3 раз и стать равной $\hbar\omega_{\mathrm{ign}}$=0,5кэВ.

Одновременно с этим, в системе координат движущейся вместе с атомами позитрония в γ раз увеличивается плотность фотонов во встречном поджигающем пучке, а угловая расходимость $\Delta\Omega$ уменьшается в $\left[\gamma + (\gamma^2 - 1)^{1/2}\right]^2 \approx 4\gamma^2$ раз. В результате, яркость (спектрально-угловая плотность потока фотонов) этого пучка фотонов возрастает в $4\gamma^3$ раз, что для приведенного численного примера составляет значительную величину 5.10^8 .

Разумеется другой пучок фотонов, совпадающий с направлением движения атомов позитрония, испытывает обратную трансформацию и, поэтому, энергия поджигающих фотонов в нем должна быть чрезвычайно большой $2\mathrm{mc}^2\gamma$ =0,5ГэВ.

Следует отметить, однако, что фотоны нужной энергии и нужного направления рождаются в каждом акте двухквантовой спонтанно-стимулированной излучательной аннигиляции атомов парапозитрония, вызванной одним только первым поджигающим пучком. В таких радиационных переходах внешнее электромагнитное излучение стимулирует лишь одну часть двухквантового перехода к испусканию фотона, второй же фотон излучается спонтанно. При этом, согласно законам сохранения энергии и импульса в системе покоя атома позитрония частоты обоих фотонов совпадают, а направление вылета спонтанного фотона строго противоположно направлению стимулирующего излучения. Рожденные таким образом спонтанные фотоны идеально подходят для последующего участия в актах двухквантовой стимулированно-стимулированной аннигиляции атомов позитрония и, следовательно, могут играть роль второго поджигающего пучка.

Необходимо подчеркнуть, что такая жесткая связь между стимулированным и спонтанным фотонами уникальна именно для процесса аннигилации, когда происходит исчезновение излучателя. При ядерных двухквантовых спонтанно-стимулированных переходах угол вылета спонтанного фотона по отношению к направлению стимулирующего излучения может изменяться в широких пределах от 0 до 2π, а импульс отдачи принимает на себя ядро.

Специфическими свойствами обладает внешний поджиг трехквантовой индуцированной аннигиляции ортопозитрония. В силу законов сохранения энергии и импульса, энергии $\hbar\omega_1, \hbar\omega_2, \hbar\omega_3$ трех испущенных аннигиляционных фотонов должны изображаться длинами сторон треугольника с периметром $2\mathrm{mc}^2$. Поэтому векторы импульсов этих фотонов и углы между ними полностью определяются заданием энергий двух фотонов. При этом, если два фотона имеют суммарную энергию $\hbar\omega_1 + \hbar\omega_2 = \mathrm{mc}^2$, то импульсы всех трех фотонов должны лежать на одной прямой и третий фотон с энергией $\hbar\omega_3 = \mathrm{mc}^2$ испускается в направлении в точности обратном направлению испускания первых двух. Для внешнего поджига “полностью” стимулированной аннигиляции теперь потребуется три пучка фотонов - два параллельных в одном направлении и один в противоположном направлении.

Особое значение имеет вырожденный случай равенства энергий $\hbar\omega_1 = \hbar\omega_2 = mc^2/2$, когда два параллельных потока фотонов одного направления сливаются в один поток, каждый фотон которого индуцирует в акте аннигиляции ортопозитрония испускание сразу двух квантов с энергией $\hbar\omega = mc^2/2$. В результате коэффициент усиления этого пучка фотонов возрастает вдвое по сравнению с пучком встречных фотонов. При этом, условия доплеровского релятивистского преобразования частоты поджигающих фотонов в этом пучке вдвое более мягкие, чем при поджиге двухквантовой индуцированной аннигиляции парапозитрония (1). Например, при энергии поджигающих фотонов $\hbar\omega_{ign}$=0,5кэВ теперь достаточно использовать релятивистский пучок атомов ортопозитрония с энергией электронов и позитронов $mc^2\gamma = 130$МэВ ($\gamma \approx 250$) против 260МэВ для двухквантовой аннигилации парапозитрония.

Непременным атрибутом стимулированной аннигиляции позитрония в поле двух встречных пучков фотонов является динамическая распределенная обратная связь между встречными волнами. Рожденные фотоны по самому смыслу индуцированного излучения оказываются безупречно сфазированными и попадают в нужную моду. Нелинейность обратной связи определяет динамику усиления встречных пучков фотонов и при определенных условиях вызывает лавинообразную индуцированную аннигиляцию атомов позитрония, которая сопровождается излучением гигантского импульса гамма квантов.

3. Динамика усиления встречных поджигающих пучков. Парапозитроний.

В системе координат, движущейся вместе с пучком атомов парапозитрония с концентрацией N, усиление встречных потоков гамма квантов с плотностю I и I^* [$см^{-2}c^{-1}$] в телесном угле $\Delta\Omega$ и спектральной полосе $\Delta\omega_0 = 1/\tau$, равной обратному времени жизни парапозитрония, описывается в стационарном случае уравнениями

$$\frac{dI}{dz} = \beta(\varepsilon N - 1)II^* + \gamma\varepsilon N(I + I^*) + \varepsilon_0\varepsilon\frac{N}{\tau} - \sigma(N_+ + N_-)I \tag{2}$$

$$-\frac{dI^*}{dz} = \beta(\varepsilon N - 1)II^* + \gamma\varepsilon N(I + I^*) + \varepsilon_0\varepsilon\frac{N}{\tau} - \sigma(N_+ + N_-)I^* \tag{3}$$

где множитель $\varepsilon = \Delta\omega_0/\Delta\omega_\mathrm{D}$ показывает долю атомов позитрония, взаимодействующих с внешними поджигающими пучками фотонов, $(\mathrm{N}_+ + \mathrm{N}_-)$ - совокупное число электронов и позитронов в пучке. Первые члены уравнений отвечают за двухквантовое стимулированное испускание или поглощение фотонов с коэффициентом β [см^{4}с] при аннигиляции или рождении атомов позитрония, вторые члены учитывают спонтанно-стимулированное испускание фотонов во встречные пучки I и I^* с коэффициентом γ [см^2], третьи члены описывают чисто спонтанное испускание, причем множитель $\varepsilon_0 = \Delta\Omega/4\pi$ показывает долю фотонов, попадающих в телесный угол $\Delta\Omega$, охватывающий пучки I и I^*, и последние члены характеризуют полные потери фотонов из каждого пучка с сечением рассеяния σ [см^2]; z - продольная координата зоны усиления с длиной L .

Положительное усиление с $\mathrm{dI}/\mathrm{dz} > 0$ и $-\mathrm{dI}^*/\mathrm{dz} > 0$ достигается если

$$2\beta(\varepsilon\mathrm{N} - 1)\mathrm{II}^* > \left[\sigma(\mathrm{N}_+ + \mathrm{N}_-) - 2\gamma\varepsilon\mathrm{N}\right](\mathrm{I} + \mathrm{I}^*) - 2\varepsilon_0\varepsilon\,\mathrm{N}/\tau \ . \tag{4}$$

Отсюда, вводя интенсивности поджигающих пучков фотонов I_ign и $\mathrm{I}^*_\mathrm{ign} = \mathrm{I}_\mathrm{ign}/\mu$ на входе в область усиления, получаем условие, которое можно рассматривать как пороговое для поджига

$$\mathrm{I}_\mathrm{ign} > \mathrm{I}_0(1+\mu)\ , \qquad \mathrm{I}^*_\mathrm{ign} > \mathrm{I}_0(1+\mu)/\mu \tag{5}$$

где

$$\mathrm{I}_0 = \frac{\sigma(\mathrm{N}_+ + \mathrm{N}_-)/\mathrm{N}_0 - 2\varepsilon\gamma}{2\varepsilon\beta} \tag{6}$$

и последний член в (4) опущен из-за малости множителя ε_0, а также предполагается, что начальная концентрация атомов позитрония N_0 достаточно велика, так что $\varepsilon N_0 >> 1$.

Если интенсивности встречных пучков существенно превышают пороговый уровень, то в уравнениях (2), (3) можно пренебречь всеми членами, кроме первых. Тогда получаем

$$\frac{dI}{dz} = -\frac{dI^*}{dz} = \beta\varepsilon N I I^* \tag{7}$$

откуда $d(I + I^*)/dz = 0$ и $I + I^* = I_{ign} + I^*_{out} = I^*_{ign} + I_{out} = \text{const}$, где I_{out} и I^*_{out} - интенсивности пучков на выходе из области усиления. Результатом интегрирования системы (7) является трансцендентное уравнение для "чистого" значения выходной интенсивности $I_n = I_{out} - I_{ign} = I^*_{out} - I^*_{ign}$ на длине усиления L

$$\left(\frac{I_n}{I_{ign}} + 1 + \mu\right)^{-1} \ln\left[\left(\frac{I_n}{I_{ign}} + 1\right)\left(\mu\frac{I_n}{I_{ign}} + 1\right)\right] = I_{ign}\beta\varepsilon\tilde{N}L \tag{8}$$

где $\tilde{N}$ - средняя концентрация атомов позитрония в пучке

$$\tilde{N} = \frac{1}{L}\int_0^L N(z)dz \tag{9}$$

Решение этого уравнения представлено на рис.1 в виде зависимости I_n/I_{ign} от параметра активности усиливающей среды $A = I_{ign}\beta\varepsilon\tilde{N}L$ для различной степени ассимметрии интенсивностей поджигающих пучков фотонов μ.

Особенностью кривых является их неоднозначность и гистерезисный характер. При достижении параметра активности A критического значения происходит лавинообразный скачок на верхний участок S - образных кривых и резкое возрастание скорости индуцированной аннигиляции, сопровождающееся излучением гигантского импульса гамма квантов.

Необходимо отметить, что разрывный характер кривых на рис.1 имеет своей причиной действие динамической распределенной обратной связи между встречными потоками фотонов [16]. Коэффициент обратной связи ρ , определяемый через приращение плотности потока фотонов обратной волны dI^* на элементе длины dz под действием прямой волны с плотностью потока I, имеет вид

$$\rho = \frac{1}{I}\frac{dI^*}{dz} = \beta\varepsilon N I^* . \tag{10}$$

Нелинейный характер связи между встречными волнами обусловленный актами стимулированно-стимулированной аннигиляции парапозитрония приводит к росту коэффициента связи ρ вместе с I^* , что и является причиной лавинообразного развития процесса индуцированной аннигиляции.

3. Вероятность индуцированной аннигиляции парапозитрония

Связь между вероятностью спонтанной и индуцированной аннигиляции парапозитрония проще всего установить с помощью соотношения Эйнштейна между спектральными коэффициентами спонтанного излучения $A(\omega)$ и индуцированного излучения $B(\omega)$ [17]

$$\frac{A}{B} = \frac{\hbar\omega^3}{\pi^2 c^3} , \tag{11}$$

где коэффициент $A(\omega)$ определяет вероятность спонтанного излучения фотона частоты ω в спектральный интервал $d\omega$ и интервал телесных углов $d\Omega$

$$dW_s = A(\omega) d\omega \frac{d\Omega}{4\pi} , \tag{12}$$

а коэффициент $B(\omega)$ - вероятность стимулированного излучения фотона той же частоты

$$dW_i = B(\omega) U(\omega,\theta,\varphi) d\omega d\Omega \tag{13}$$

в присутствии электромагнитной волны со спектрально-угловой плотностью энергии $U(\omega,\theta,\varphi)$. С помощью соотношения Эйнштейна (11) перепишем (13) в виде удобном для дальнейшего применения

$$dW_i = (\lambda^2/4)A(\omega)I(\omega,\mathbf{k})d\omega d\Omega \qquad (14)$$

где $I(\omega,\mathbf{k}) = I(\omega,\theta,\varphi)$ - спектрально-угловая плотность потока фотонов (яркость) стимулирующего излучения с длиной волны λ и волновым вектором $\mathbf{k}$.

Вероятность в единицу времени двухквантовой спонтанно-спонтанной аннигиляции атома парапозитрония с излучением фотона частоты ω в спектральный интервал $d\omega$ и интервал телесных углов $d\Omega$ равна

$$dW_{ss} = W_{ss}g(\omega-\omega_0)d\omega\frac{d\Omega}{4\pi}, \qquad (15)$$

где $W_{ss} = 1/\tau$ - обратное время жизни парапозитрония, $W_{ss} = \alpha^5 mc^2/(2\hbar)$ = 0,8.10^{10} c^{-1} [11], $g(\omega)$ - функция Лоренца с шириной $\Delta\omega_0 = 1/\tau$ и $\hbar\omega_0 = mc^2$.

Используя дважды переход от (12) к (14), найдем сначала вероятность в единицу времени спонтанно-стимулированной аннигиляции

$$dW_{is} = (\lambda^2/4)W_{ss}g(\omega-\omega_0)I(\omega,\mathbf{k})d\omega d\Omega, \qquad (16)$$

а затем и стимулированно-стимулированной аннигиляции парапозитрония

$$dW_{ii} = (\pi\lambda^4/4)W_{ss}g(\omega-\omega_0)I(\omega,\mathbf{k})I^*(\omega,-\mathbf{k})d\omega d\Omega \qquad (17)$$

во встречных потоках стимулирующего излучения с яркостью фотонных пучков $I(\omega,\mathbf{k})$ и $I^*(\omega,-\mathbf{k})$.

Соответственно, скоростная константа γ в уравнениях (2), (3), отвечающая за вклад спонтанно-стимулированной аннигиляции, равна

$$\gamma = \lambda_C^2/(2\pi), \qquad (18)$$

где $\lambda_C = 2\pi\hbar/(mc)$=2,4.$10^{-10}$см есть комптоновская длина волны и $\gamma = 0{,}9.10^{-20}$см^2, а скоростная константа β , дающая вклад стимулированно-стимулированных актов аннигиляции имеет вид

$$\beta = \frac{\lambda_C^4}{2}\frac{1}{\Delta\omega_0\Delta\Omega}. \tag{19}$$

4. Численные оценки для парапозитрония

Численные оценки представлены для пучка, в котором атомов парапозитрония в 10^2 раз меньше, чем свободных электронов и позитронов, а степень монокинетичности такова, что доля атомов парапозитрония, взаимодействующих с фотонными пучками, составляет $\varepsilon = 10^{-5}$.

Пороговые яркости поджигающих пучков фотонов оцениваются по (5) и (6) величинами $I_{ign}/(\Delta\omega_0\Delta\Omega) > 8.10^{22}$ см$^{-2}$ страд$^{-1}$ и $I^*_{ign}/(\Delta\omega_0\Delta\Omega) > 8.10^{20}$ см$^{-2}$ страд$^{-1}$, где принят коэффициент ассимметрии μ=100 и учтено, что основной причиной потерь гамма квантов из пучка является комптоновское рассеяние фотонов с сечением в рассматриваемой области частот равным [11] $\sigma \approx 0{,}4(8\pi/3)r_0^2 =$ 2,6.10^{-25} см^2 (r_0 - классический радиус электрона, r_0=2,8.10^{-13}см).

Параметр активности среды, критический для начала поджига лавинообразной индуцированной аннигиляции парапозитрония определяется условием $A_{cr} = \left(I_{ign}\beta\varepsilon\tilde{N}L\right)_{cr} \sim 1$ (см. рис.1), которое для порогового значения яркости поджигающего фотонного пучка $I_{ign}/(\Delta\omega_0\Delta\Omega) = 8.10^{22}$ см$^{-2}$ страд$^{-1}$ может быть удовлетворено, например, при $\tilde{N} = 10^{18}$см$^{-3}$ и $L = 10$м . Критические значения A_{cr} и $\left(I_n/I_{ign}\right)_{cr}$ логарифмически слабо зависят от степени ассимметрии поджига μ , что позволяет надеяться на старт процесса лавинообразной индуцированной аннигиляции позитрония с уровня плотности потока спонтанных фотонов, испущенных при спонтанно-стимулированных актах аннигиляции, то есть в отсутствие внешнего поджигающего пучка I^*. В этом случае имеет смысл использовать преимущества

релятивистского движения атомов позитрония. Как указывалось выше, встречное движение атомов позитрония в пучке с релятивистским фактором $\gamma \approx 500$ позволяет в $4\gamma^3$=5.10^8 раз снизить яркость поджигающего фотонного пучка. Кроме того, энергия фотонов в поджигающем пучке может быть снижена, согласно (1), в 2γ=10^3 раз. В итоге, пороговое значение спектрально-угловой плотности потока энергии в поджигающем пучке составит 0,8.10^{17} эВ см$^{-2}$ страд$^{-1}$, что попадает в диапазон достижимый с помощью рентгеновских лазеров.

5. Динамика усиления встречных поджигающих пучков. Ортопозитроний.

Лавинообразное излучение гигантского импульса гамма квантов происходит и при внешнем поджиге трехквантовой индуцированной аннигиляции ортопозитрония. В вырожденном случае, когда поджиг осуществляется двумя встречными пучками фотонов с плотностями потоков I и I^* и энергиями фотонов,соответственно, $\hbar\omega = \hbar\omega_0/2 = \mathrm{mc}^2/2$ и $\hbar\omega^* = \hbar\omega_0 = \mathrm{mc}^2$, стационарный процесс усиления описывается системой из двух уравнений

$$\frac{1}{2}\frac{d\mathrm{I}}{dz} = -\frac{d\mathrm{I}^*}{dz} = \chi\varepsilon \mathrm{N}\mathrm{I}^2\mathrm{I}^*. \tag{20}$$

Интегрирование этой системы приводит к трансцендентному уравнению

$$\frac{\ln\left[\left(\mu\frac{\mathrm{I_n}}{\mathrm{I_{ign}}}+1\right)\left(2\frac{\mathrm{I_n}}{\mathrm{I_{ign}}}+1\right)\right]}{\left(2\frac{\mathrm{I_n}}{\mathrm{I_{ign}}}+1+\frac{2}{\mu}\right)^2} + \frac{2\frac{\mathrm{I_n}}{\mathrm{I_{ign}}}}{\left(2\frac{\mathrm{I_n}}{\mathrm{I_{ign}}}+1+\frac{2}{\mu}\right)\left(2\frac{\mathrm{I_n}}{\mathrm{I_{ign}}}+1\right)} = \mathrm{I}_{\mathrm{ign}}^2\chi\varepsilon\tilde{\mathrm{N}}\mathrm{L} \tag{21}$$

где $\mathrm{I_n} = \mathrm{I}^*_{\mathrm{ign}} - \mathrm{I}^*_{\mathrm{out}} = (1/2)(\mathrm{I_{ign}} - \mathrm{I_{out}})$ - чистый выход пучка I^*, а $\mathrm{I_{ign}}$ и $\mathrm{I}^*_{\mathrm{ign}} = \mathrm{I_{ign}}/\mu$ плотности внешних потоков поджигающих фотонов на входе в область усиления. Решение этого уравнения представлено на рис.2 в виде зависимости $\mathrm{I_n}/\mathrm{I_{ign}}$ от параметра активности среды $\mathrm{A} = \mathrm{I}_{\mathrm{ign}}^2\chi\varepsilon\tilde{\mathrm{N}}\mathrm{L}$ при различных значениях ассимметрии интенсивностей поджигающих пучков. Видна неоднозначность этой зависимости,

свидетельствующая о лавинообразном развитии процесса индуцированной аннигиляции при критических значениях параметра активности $A = A_{cr} \sim 1$.

Скоростная константа χ , входящая в уравнения (20), может быть получена следующим образом. Начать надо с выражения для вероятности в единицу времении спонтанной трехквантовой аннигиляции с испусканием двух фотонов частоты $\omega = \omega_0/2$ в одном направлении и третьего фотона частоты $\omega^* = \omega_0$ в противоположном направлении в частотный интервал $d\omega$ и телесный угол $d\Omega$

$$dW_s^{(0)} = W_s^{(0)} g(\omega - \frac{\omega_0}{2}) d\omega \frac{d\Omega}{4\pi} , \qquad (22)$$

где $W_s^{(0)}$ связана с обратным временем жизни ортопозитрония

$$\frac{1}{\tau_0} = \frac{2(\pi^2 - 9)}{9\pi} \alpha^6 \frac{mc^2}{\hbar} = 0,7.10^7 c^{-1} \qquad (23)$$

следующим соотношением [11]

$$W_s^{(0)} = \frac{2}{(\pi^2 - 9)} \frac{1}{\omega_0^2 \tau_0^3} . \qquad (24)$$

Применяя затем трижды переход от (12) к (14), придем к выражению для скорости “полностью” индуцированной трехквантовой аннигиляции ортопозитрония во встречных фотонных пучках

$$dW_i^{(0)} = \left(\frac{\pi\lambda^3}{4} \right)^2 W_s^{(0)} g(\omega - \frac{\omega_0}{2}) I^2(\omega, \mathbf{k}) I^*(2\omega, -2\mathbf{k}) d\omega d\Omega , \qquad (25)$$

откуда для скоростной константы χ индуцированной аннигиляции ортопозитрония получаем следующее выражение

$$\chi = \frac{4}{\pi(\pi^2 - 9)} \frac{\lambda_C^8}{\tau_0^2 c^2} \frac{1}{(\Delta\omega_0 \Delta\Omega)^2} . \qquad (26)$$

Оценка пороговой плотности потока фотонов поджигающего пучка приводит к

$$I_{ign} > \left[\frac{\sigma(N_+ + N_-)}{N} \frac{(\mu+1)}{\varepsilon\chi} \right]^{1/2}, \tag{27}$$

что дает чрезвычайно большую пороговую яркость $I_{ign}/(\Delta\omega_0 \Delta\Omega) = 1{,}5.10^{34}$ см$^{-2}$ страд$^{-1}$, далеко выходящую за пределы возможностей современных источников излучения и делающую поджиг индуцированной аннигиляции ортопозитрония в настоящее время нереальным.

6. Заключение

Проведенный анализ раскрывает основные преимущества и недостатки метода внешнего поджига встречными фотонными пучками процесса индуцированной аннигиляции атомов позитрония:

- Устанавливается присущий лишь двухквантовому стимулированному испусканию во встречных пучках специальный вид динамической распределенной обратной связи без каких-либо отражающих структур.
- Нелинейность обратной связи с коэффициентом, пропорциональным интенсивности пучка фотонов, вызывает лавинообразную аннигиляцию атомов позитрония, сопровождающуюся излучением гигантского импульса гамма квантов.
- Возможность использования релятивистских пучков атомов позитрония существенно снижает требования к источнику поджигающих фотонов встречного направления. При энергии электронов и позитронов $mc^2\gamma \approx 260$ Мэв ($\gamma \approx 500$) пороговая спектрально-угловая плотность потока энергии в поджигающем пучке составляет для парапозитрония величину $0{,}8.10^{17}$ эВ см$^{-2}$ страд$^{-1}$, попадающую в диапазон рентгеновских лазеров.
- Спонтанные фотоны, испускаемые в каждом акте спонтанно-стимулированной излучательной аннигиляции атомов позитрония, вызванной одним только первым поджигающим пучком, идеально подходят для последующего участия в актах двухквантовой стимулированно-стимулированной аннигиляции и, следовательно, могут играть роль второго поджигающего пучка.

- Осуществлению подобного процесса сегодня мешает отсутствие источников поджигающих гамма квантов достаточной яркости (для ортопозитрония) и достаточной длительности импульса (для парапозитрония) и поэтому преимущества метода внешенго поджига индуцированной аннигиляции атомов позитрония могут, по-видимому успешно проявиться лишь в оконечной ступени источника гамма-квантов (например, после рентгеновского или гамма лазера, релятивистского ондулятора или лазера на свободных электронах и т. п.) для получения кратковременного импульса гамма фотонов значительной пиковой амплитуды.

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